Accurate Wide-Range Design Equations for the Frequency-Dependent Characteristic of Parallel Coupled Microstrip Lines

MANFRED KIRSCHNING AND ROLF H. JANSEN, MEMBER, IEEE

Abstract —**In this paper, closed-form expressions are presented which model the frequency-dependent even- and odd-mode characteristics of parallel coupled microstrip lines with hitherto unattained accuracy and range of validity. They include the effective dielectric constants, the characteristic impedances using the power–current formulation, as well as the open-end equivalent lengths for the two fundamental modes on coupled microstrip. The formulas are accurate into the millimeter-wave region. They are based on an extensive set of accurate numerical data which were generated by a rigorous spectral-domain hybrid-mode approach and are believed to represent a substantial improvement compared to the state-of-the-art and with respect to the computer-aided design of coupled microstrip filters, directional couplers, and related components.**

I. Introduction

THE STATIC NUMERICAL solution of Bryant and Weiss in 1968 [1] provided one of the first reliable and accurate sources of information on coupled microstrip transmission-line characteristics. Their MSTRIP computer program [2] has been used by numerous authors as a reference, and has been validated by comparison with many other sources. Several years after the appearance of this computer program, the first frequency-dependent spectral domain analyses of coupled microstrip lines became available [3]–[7]. Today, the algorithms used to perform such computations have been developed to a higher maturity level, and efficient program packages exist which are suited for industrial application [8]–[13]. Using these, the frequency-dependent characteristics of coupled microstrip lines can be calculated to practically any required degree of accuracy.

Parallel to the mentioned rigorous computational efforts, a larger number of contributions have dealt with the description of coupled microstrip design data in the form of closed analytical expressions. A selection of the more representative papers on this topic is given in the references of this paper [14]–[25]. Since reviews of the state-of-the-art of coupled microstrip design formulas, up to about the end of 1979, can be found in three recent books on computer-aided microstrip circuit design [26]–[28], only the latest developments have to be recapitulated here. The accumulation of analytical approaches to the problem of describing coupled microstrips within the last few years can partially be explained as a consequence of microwave technology improvements, in so far as circuits and substrates of decreased tolerances justify and inspire descriptions of increased accuracy. These descriptions should preferrably be available as closed-form analytical models which are a requirement resulting from the growing application of computer-aided design tools in the microwave industry. Numerical algorithms, like those listed in [1]–[13], are accurate and are reliable sources of design information, but are too time-consuming for direct use in circuit optimization routines. Finally, the recent trend toward analytical modeling efforts on the whole has been accelerated by the rapid development of monolithic microwave integrated circuits during the last time period.

Today, it appears that the most accurate and generally valid static model of coupled microstrips has been given by Hammerstad and Jensen in 1980 [24]. The goal of these authors was to obtain results with errors at least less than those caused by physical tolerances. Recently March [25] verified the accuracy specifications of [24] through detailed comparison with the MSTRIP computer program [2] and the results reported by Jansen [10]. Also, comparison with the spectral-domain hybrid-mode program used in this paper [11], [13] confirms that the static equations presented by Hammerstad and Jensen generally have maximum errors less than 1 percent, except for some limiting situations. In addition, the computer program used here as a reference has been validated itself extensively by single and coupled microstrip dispersion measurements since its generation in 1978. Therefore, Hammerstad and Jensen's static equations can be judged to be superior to those of Garg and Bahl [22], who report a 3-percent (characteristic impedances) and 4-percent (effective dielectric constants) accuracy for their static semiempirical coupled microstrip expressions. Furthermore, the formulas of [24] are valid in an extended range of geometrical parameters and are essentially constructed to incorporate the correct asymptotic behavior with respect to these.

Nevertheless, there still exists no accurate frequency-

Manuscript received April 26, 1983; revised August 4, 1983.

M. Kirschning was with the University of Duisburg, Department of Electrical Engineering, FB9/ATE, Bismarckstr. 81, D-4100 Duisburg 1, West Germany. He is now with Honeywell GmbH, P.O. Box 1109, D-6457, Maintal, West Germany.

R. H. Jansen is with the University of Duisburg, Department of Electrical Engineering, FB9/ATE, Bismarckstr. 81, D-4100 Duisburg 1, West Germany.

0018-9480/84/0100-0083$01.00

dependent model for parallel coupled microstrip lines [24], [27]. Getsinger's dispersion model [15] can be shown to be asymptotically wrong for extreme values of gap width [24], and some of its limitations were reported already in 1973 [16]. This model gives relatively good results for alumina substrates [22], [25] for which it has been adjusted. However, it becomes inaccurate when a wide range of geometrical and substrate parameters is considered. In our experience, this limitation not only exists with respect to the modal effective dielectric constants, but also for the even- and odd-mode characteristic impedances of coupled microstrip lines. Furthermore, the ability of even a slightly modified version of Getsinger's dispersion model to describe the frequency dependence of coupled microstrip characteristic impedances with fair accuracy has again been demonstrated only for alumina [22], [25], and with respect to the voltage–current definition of impedance used in [10]. Theoretical and experimental work performed very recently indicates that the power–current formulation of characteristic impedance with its smaller frequency dependence should be preferred for microstrip computer-aided design [29]–[31]. Therefore, a thorough modification of Getsinger's coupled microstrip expressions as invoked in recent publications [24], [27] is performed here to provide accurate wide-range frequency-dependent design formulas. In addition, further refinement of part of the static equations of Hammerstad and Jensen [24] is performed in order to achieve better error margins as a start for wide-range modeling of dispersion. This paper sets forth a modeling approach previously applied to single microstrip lines [31]–[33]. For completeness, it includes formulas for the even- and odd-mode open-end equivalent lengths of the coupled microstrip section. The necessary reference data for these come from a three-dimensional hybrid-mode approach developed by Jansen [34], [35], which was meanwhile verified by Hornsby's results [36]. Therefore, the whole set of equations presented here is based upon reliable, validated, and cross-referenced hybrid-mode data.

II. Dispersive Coupled Microstrip Model

The analytical expressions which follow describe the effective dielectric constants, the power–current characteristic impedances, and the equivalent open-end lengths of coupled microstrip lines. The named quantities are functions of the substrate dielectric constant ϵ_r, the coupled microstrip cross-sectional geometry as depicted in Fig. 1, and are functions of frequency, except for the modal open-end length Δl_e and Δl_o. The latter are relatively small quantities and do not vary with frequency to a considerable degree in the usual range of applications up to about 18 GHz [34], [35].

The expressions given here have all been derived by successive computer matching to converged numerical results stemming from a rigorous spectral-domain hybrid-mode approach [11], [13], and [34]. The accuracies specified for them are with respect to the numerical data basis which, typically, consisted of several thousand test values.

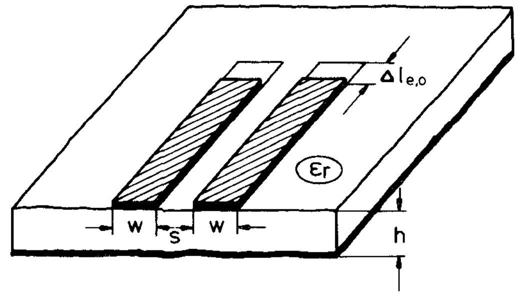


Fig. 1. Pictorial representation of coupled microstrip cross-sectional geometry and even- and odd-mode open-end equivalent lengths.

The range of validity to which the given accuracies apply is

$$0.1 \leqslant u \leqslant 10 \qquad 0.1 \leqslant g \leqslant 10 \qquad 1 \leqslant \epsilon_r \leqslant 18 \tag{1}$$

where $u = w/h$ denotes normalized strip width and $g = s/h$ is the normalized line spacing. This covers the typical range of technically meaningful parameters. Since the correct asymptotic behavior, with respect to the physical parameters, has been incorporated into the equations described here, the equations can even be applied beyond the limits of range (1), although with reduced accuracy. This is also true with respect to the frequency dependence, which is introduced here using the normalized quantity f_n, where

$$f_n = (\mathrm{f/GHz}) \cdot (\mathrm{h/mm}) \tag{2}$$

and the upper limit of which will be given separately for each set of expressions. The influence of a conducting cover plate or of lateral shielding walls is not taken into account in the derived expressions, i.e., the shielding is assumed to be far enough and of negligible effect on the line parameters. This is in agreement with usual microstrip circuit design practice. If corrections for small cover height appear to become necessary in a design, the correction formulas given by March [25] can be applied. Also, the finite thickness of the strip conductors is not accounted for in the formulas listed below. If conclusion of its effect is desired, for example, in the case of very small widths w or small spacings s, it is proposed to proceed in the way outlined in previous publications [10], [22], and [25]. The width correction introduced by Hammerstad and Jensen [24] is not recommended, since this exaggerates the effect of finite thickness on the characteristics; probably, there is a misprint in the named reference. Starting with the modeling process, an accuracy check of the static model of Hammerstad and Jensen [24] with respect to the reference basis used here [11], [13] shows that the static even-mode effective dielectric constant of [24], denoted by $\epsilon_{\mathrm{eff}_e}(0)$, is in error by not more than 0.7 percent for the range of applicability (1), while the corresponding odd-mode value, $\epsilon_{\mathrm{eff}_o}(0)$, is in error by about 4 percent in the case of low dielectric constants and large widths and spacings. For this reason, only the static value of $\epsilon_{\mathrm{eff}_e}(0)$ has been adopted here from [24] and is rewritten as

$$\epsilon_{\mathrm{eff}_e}(0) = 0.5(\epsilon_r + 1) + 0.5(\epsilon_r - 1) \cdot (1 + 10/v)^{-a_e(v) \cdot b_e(\epsilon_r)}$$

$$v = u(20+g^2)/(10+g^2)+g\cdot\exp(-g)$$
$$a_e(v) = 1+\ln\left(\left(v^4+(v/52)^2\right)/(v^4+0.432)\right)/49 + \ln\left(1+(v/18.1)^3\right)/18.7$$
$$b_e(\epsilon_r) = 0.564((\epsilon_r-0.9)/(\epsilon_r+3))^{0.053}. \quad (3)$$

In contrast to this, the static odd-mode parameter $\epsilon_{\mathrm{eff}_o}(0)$ has been remodeled for an improved accuracy of 0.5 percent over the range (1) and is given by

$$\epsilon_{\mathrm{eff}_o}(0) = (0.5(\epsilon_r+1)+a_o(u,\epsilon_r)+\epsilon_{\mathrm{eff}}(0))\cdot\exp(-c_o\cdot g^{d_o})+\epsilon_{\mathrm{eff}}(0)$$
$$a_o(u,\epsilon_r) = 0.7287(\epsilon_{\mathrm{eff}}(0)-0.5(\epsilon_r+1))\cdot(1-\exp(-0.179u))$$
$$b_o(\epsilon_r) = 0.747\epsilon_r/(0.15+\epsilon_r)$$
$$c_o = b_o(\epsilon_r)-(b_o(\epsilon_r)-0.207)\cdot\exp(-0.414u)$$
$$d_o = 0.593+0.694\cdot\exp(-0.562u) \quad (4)$$

where the quantity $\epsilon_{\mathrm{eff}}(0)$ without additional subscript refers to zero-thickness single microstrip [24] of width w. Frequency dispersion is introduced for both modes in the same form as has been done previously for single microstrip [33], namely by

$$\epsilon_{\mathrm{eff}_{e,o}}(f_n) = \epsilon_r-(\epsilon_r-\epsilon_{\mathrm{eff}_{e,o}}(0))/(1+F_{e,o}(f_n)). \quad (5)$$

This is a generalization of Getsinger's dispersion relation [15] and incorporates a more complicated form of frequency dependence in the terms denoted by $F_e(f_n)$ and $F_o(f_n)$. For the even mode on a coupled microstrip, this results in

$$F_e(f_n) = P_1P_2((P_3P_4+0.1844P_7)\cdot f_n)^{1.5763} \quad (6)$$

with

$$P_1 = 0.27488+\left(0.6315+0.525/(1+0.0157f_n)^{20}\right)\cdot u - 0.065683\cdot\exp(-8.7513u)$$
$$P_2 = 0.33622\cdot(1-\exp(-0.03442\epsilon_r))$$
$$P_3 = 0.0363\cdot\exp(-4.6u)\cdot\left(1-\exp\left(-(f_n/38.7)^{4.97}\right)\right)$$
$$P_4 = 1+2.751\cdot\left(1-\exp\left(-(\epsilon_r/15.916)^8\right)\right)$$
$$P_5 = 0.334\cdot\exp\left(-3.3(\epsilon_r/15)^3\right)+0.746$$
$$P_6 = P_5\cdot\exp\left(-(f_n/18)^{0.368}\right)$$

and

$$P_7 = 1+4.069P_6g^{0.479}\exp(-1.347g^{0.595}-0.17g^{2.5}).$$

For the odd-mode effective dielectric constant, the effect of dispersion is described by

$$F_o(f_n) = P_1P_2\cdot((P_3P_4+0.1844)\cdot f_n\cdot P_{15})^{1.5763} \quad (7)$$

with

$$P_8 = 0.7168(1+1.076/(1+0.0576(\epsilon_r-1)))$$
$$P_9 = P_8-0.7913\cdot\left(1-\exp\left(-(f_n/20)^{1.424}\right)\right)\cdot\arctan\left(2.481(\epsilon_r/8)^{0.946}\right)$$
$$P_{10} = 0.242\cdot(\epsilon_r-1)^{0.55}$$
$$P_{11} = 0.6366\cdot(\exp(-0.3401f_n)-1)\cdot\arctan\left(1.263(u/3)^{1.629}\right)$$
$$P_{12} = P_9+(1-P_9)/(1+1.183u^{1.376})$$
$$P_{13} = 1.695\cdot P_{10}/(0.414+1.605P_{10})$$
$$P_{14} = 0.8928+0.1072\cdot\left(1-\exp\left(-0.42(f_n/20)^{3.215}\right)\right)$$
$$P_{15} = \mathrm{abs}\left(1-0.8928(1+P_{11})P_{12}\cdot\exp(-P_{13}\cdot g^{1.092})/P_{14}\right).$$

The upper frequency limit of these expressions, in conjunction with the range of applicability (1), is $f_n = 25$, i.e., 25 GHz for substrates of 1-mm thickness, and even higher for thinner substrates. Within this limit, the maximum error involved is not greater than 1.4 percent. Additional tests have shown that this error does not exceed if, for example, dielectric constants near $\epsilon_r = 12.9$ and a normalized frequency of $f_n = 30$ are considered. Note, however, that these equations describe only undisturbed coupled microstrip lines, and other limitations might become effective in a circuit.

For the coupled microstrip characteristic impedances, again the attempt was made to use Hammerstad and Jensen's static formulas [24] as a starting point. However, detailed test computations revealed that the error in these, as compared to the hybrid-mode computer program employed here [11], [13], increases to about 1.5 percent for special parameter combinations near the limits of (1). Therefore, their usage would present problems, in so far as they would not provide sufficient error margin for the later inclusion of frequency dependence. So, for the static values of the even- and odd-mode characteristic impedances of coupled microstrip lines further improved expressions have been derived. Specifically, for the even mode, the static characteristic impedance is

$$Z_{L_e}(0) = Z_L(0)\cdot(\epsilon_{\mathrm{eff}}(0)/\epsilon_{\mathrm{eff}_e}(0))^{0.5}\cdot 1/\left(1-(Z_L(0)/377\Omega)\cdot(\epsilon_{\mathrm{eff}}(0))^{0.5}\cdot Q_4\right) \quad (8)$$

with

$$Q_1 = 0.8695\cdot u^{0.194}$$
$$Q_2 = 1+0.7519g+0.189\cdot g^{2.31}$$
$$Q_3 = 0.1975+\left(16.6+(8.4/g)^6\right)^{-0.387} + \ln\left(g^{10}/(1+(g/3.4)^{10})\right)/241$$

$$Q_4 = (2Q_1/Q_2) \cdot \left(\exp(-g)\cdot u^{Q_3} + (2-\exp(-g))\cdot u^{-Q_3}\right)^{-1}.$$

The quantities without the subscript e in the main expression are again those for a zero-thickness single microstrip [24] of width w. Similarly, the odd-mode impedance is written

$$Z_{L_o}(0) = Z_L(0)\cdot\left(\epsilon_{\text{eff}}(0)/\epsilon_{\text{eff}_o}(0)\right)^{0.5} \cdot 1/\left(1-\left(Z_L(0)/377\Omega\right)\cdot\left(\epsilon_{\text{eff}}(0)\right)^{0.5}\cdot Q_{10}\right) \quad (9)$$

with

$$Q_5 = 1.794 + 1.14\cdot\ln\left(1+0.638/(g+0.517g^{2.43})\right)$$

$$Q_6 = 0.2305 + \ln\left(g^{10}/\left(1+(g/5.8)^{10}\right)\right)/281.3 + \ln\left(1+0.598g^{1.154}\right)/5.1$$

$$Q_7 = (10+190g^2)/(1+82.3g^3)$$

$$Q_8 = \exp\left(-6.5-0.95\ln(g)-(g/0.15)^5\right)$$

$$Q_9 = \ln(Q_7)\cdot(Q_8+1/16.5)$$

$$Q_{10} = Q_2^{-1}\cdot\left(Q_2Q_4 - Q_5\cdot\exp\left(\ln(u)\cdot Q_6\cdot u^{-Q_9}\right)\right).$$

The accuracy of these new static expressions is better than 0.6 percent for both modes in the range of validity (1). Considering the frequency dependence of the characteristic impedances, the power–current formulation prevails by analogy to the treatment of a single microstrip [31]. Impedance dispersion, as resulting from numerical hybrid-mode computations [11], [13], is included for the even mode in the form

$$Z_{L_e}(f_n) = Z_{L_e}(0)\cdot\left(0.9408\left(\epsilon_{\text{eff}}(f_n)\right)^{C_e}-0.9603\right)^{Q_o} \cdot 1/\left((0.9408-d_e)\left(\epsilon_{\text{eff}}(f_n)\right)^{C_e}-0.9603\right)^{Q_o} \quad (10)$$

with

$$C_e = 1+1.275\left(1-\exp\left(-0.004625p_e\epsilon_r^{1.674}\cdot(f_n/18.365)^{2.745}\right)\right) - Q_{12}+Q_{16}-Q_{17}+Q_{18}+Q_{20}$$

$$d_e = 5.086q_e\cdot\left(r_e/(0.3838+0.386q_e)\right)\cdot\left(\exp(-22.2u^{1.92})/(1+1.2992r_e)\right)\cdot\left((\epsilon_r-1)^6/\left(1+10(\epsilon_r-1)^6\right)\right)$$

$$p_e = 4.766\cdot\exp(-3.228\cdot u^{0.641})$$

$$q_e = 0.016+(0.0514\epsilon_r\cdot Q_{21})^{4.524}$$

$$r_e = (f_n/28.843)^{12}$$

and

$$Q_{11} = 0.893\cdot\left(1-0.3/\left(1+0.7(\epsilon_r-1)\right)\right)$$

$$Q_{12} = 2.121\left((f_n/20)^{4.91}/\left(1+Q_{11}\cdot(f_n/20)^{4.91}\right)\right)\cdot\exp(-2.87g)\cdot g^{0.902}$$

$$Q_{13} = 1+0.038(\epsilon_r/8)^{5.1}$$

$$Q_{14} = 1+1.203(\epsilon_r/15)^4/\left(1+(\epsilon_r/15)^4\right)$$

$$Q_{15} = 1.887\cdot\exp\left(-1.5g^{0.84}\right)\cdot g^{Q_{14}}\cdot\left(1+0.41(f_n/15)^3\cdot u^{2/Q_{13}}/\left(0.125+u^{1.626/Q_{13}}\right)\right)^{-1}$$

$$Q_{16} = \left(1+9/\left(1+0.403(\epsilon_r-1)^2\right)\right)\cdot Q_{15}$$

$$Q_{17} = 0.394\cdot\left(1-\exp\left(-1.47(u/7)^{0.672}\right)\right)\cdot\left(1-\exp\left(-4.25(f_n/20)^{1.87}\right)\right)$$

$$Q_{18} = 0.61\cdot\left(1-\exp\left(-2.13(u/8)^{1.593}\right)\right)/\left(1+6.544g^{4.17}\right)$$

$$Q_{19} = 0.21g^4\left((1+0.18g^{4.9})\cdot(1+0.1u^2)\left(1+(f_n/24)^3\right)\right)^{-1}$$

$$Q_{20} = \left(0.09+1/\left(1+0.1(\epsilon_r-1)^{2.7}\right)\right)\cdot Q_{19}$$

$$Q_{21} = \text{abs}\left(1-42.54g^{0.133}\cdot\exp(-0.812g)\cdot u^{2.5}/(1+0.033u^{2.5})\right).$$

In the above equations, $\epsilon_{\text{eff}}(f_n)$ denotes the single microstrip effective dielectric constant as described in [33] as a function of frequency. The auxiliary quantity Q_o also refers to single microstrips. It is the exponential term which appears in the description of single-line impedance dispersion in [31, eq. (5)] and is denoted by R_{17} there. Since it consists of a chain of several expressions, it is not repeated explicitly here. Instead, it is recommended that the previously given single-line expressions [31]–[33] and the equations presented here be used for implementation on a desktop computer as a whole. For the odd-mode on coupled microstrip lines, the frequency-dependent characteristic impedance is modeled by

$$Z_{L_o}(f_n) = Z_L(f_n) + \left(Z_{L_o}(0)\cdot\left(\epsilon_{\text{eff}_o}(f_n)/\epsilon_{\text{eff}_o}(0)\right)^{Q_{22}} - Z_L(f_n)Q_{23}\right)\cdot\left(1+Q_{24}+(0.46g)^{2.2}\cdot Q_{25}\right)^{-1}$$

with

$$Q_{22} = 0.925(f_n/Q_{26})^{1.536}/\left(1+0.3(f_n/30)^{1.536}\right)$$

$$Q_{23} = 1+0.005f_n\cdot Q_{27}\cdot\left(\left(1+0.812(f_n/15)^{1.9}\right)\cdot(1+0.025u^2)\right)^{-1}$$

$$Q_{24} = 2.506Q_{28}\cdot u^{0.894}\cdot\left((1+1.3u)f_n/99.25\right)^{4.29}\cdot(3.575+u^{0.894})^{-1}$$

$$Q_{25} = \left(0.3f_n^2/(10+f_n^2)\right)\cdot\left(1+2.333(\epsilon_r-1)^2/\left(5+(\epsilon_r-1)^2\right)\right)$$

$$Q_{26} = 30-22.2\left(\left((\epsilon_r-1)/13\right)^{12}/\left(1+3\left((\epsilon_r-1)/13\right)^{12}\right)\right) - Q_{29}$$

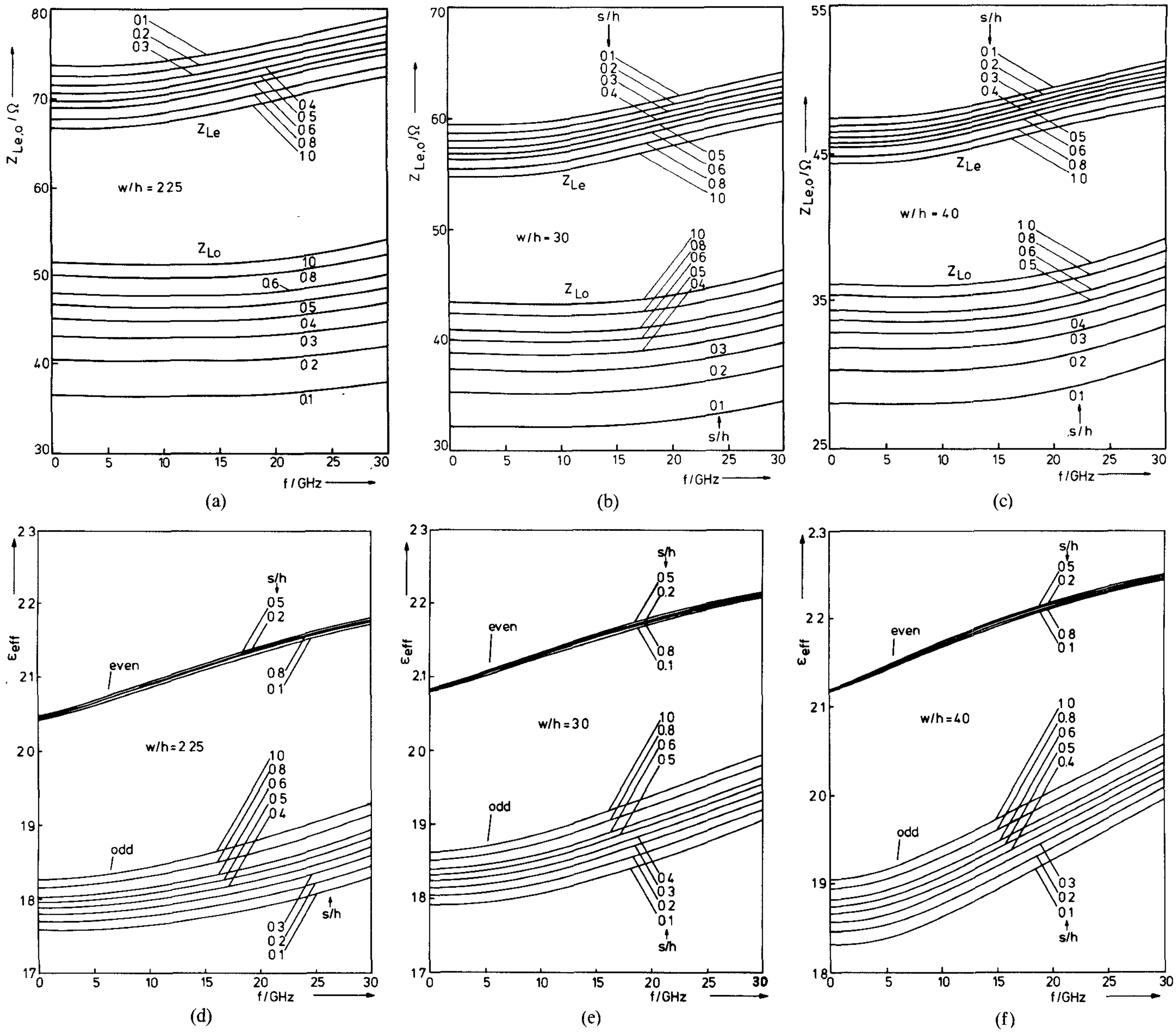


Fig. 2. The frequency-dependent even- and odd-mode effective dielectric constants and characteristic impedances of coupled microstrip lines on a plastic substate (RT-Duroid 5870, $\epsilon_r = 2.35$, $h = 0.79$ mm). (a), (d) $w/h = 2.25$. (b), (e) $w/h = 3.0$. (c), (f) $w/h = 4.00$.

$$Q_{27} = 0.4g^{0.84} \cdot \left(1 + 2.5(\epsilon_r - 1)^{1.5}/\left(5 + (\epsilon_r - 1)^{1.5}\right)\right)$$

$$Q_{28} = 0.149(\epsilon_r - 1)^3/\left(94.5 + 0.038(\epsilon_r - 1)^3\right)$$

$$Q_{29} = 15.16/\left(1 + 0.196(\epsilon_r - 1)^2\right).$$

The quantity $Z_L(f_n)$ is the frequency-dependent power–current characteristic impedance formulation of a single microstrip with width w [31]. The range of applicability (1) applies again, and the impedance equations (10) and (11) are valid up to $f_n = 20$, with a maximum error smaller than 2.5 percent. If the specified upper value of the substrate dielectric constant is reduced from 18 to 12.9, the expressions can be used up to $f_n = 25$ without a decrease in accuracy. This would correspond to about 40 GHz for a 25-mil-thick substrate. Modeling accuracy is typically better than 1.5 percent if usage is restricted to $\epsilon_r \leqslant 12.9$ and $f_n \leqslant 15$. As outlined for the effective dielectric constants, additional limitations may have to be regarded in an actual microstrip circuit.

Pictorial representation of (5)–(7), and (10), (11) is given in Figs. 2(a)–(f) and 3(a)–(f) for two widely used, commercially available substrates, namely RT-Duroid 5870 ($\epsilon_r = 2.35$, $h = 0.79$ mm) and alumina ($\epsilon_r = 9.70$, $h = 0.64$ mm). These are included as an immediate design aid and as a reference for the installation of the formulas on a computer. The line widths in Figs. 2 and 3 were chosen so that the equivalent single microstrip impedances, i.e., those for very loose coupling, are grouped around 50 Ω. With the

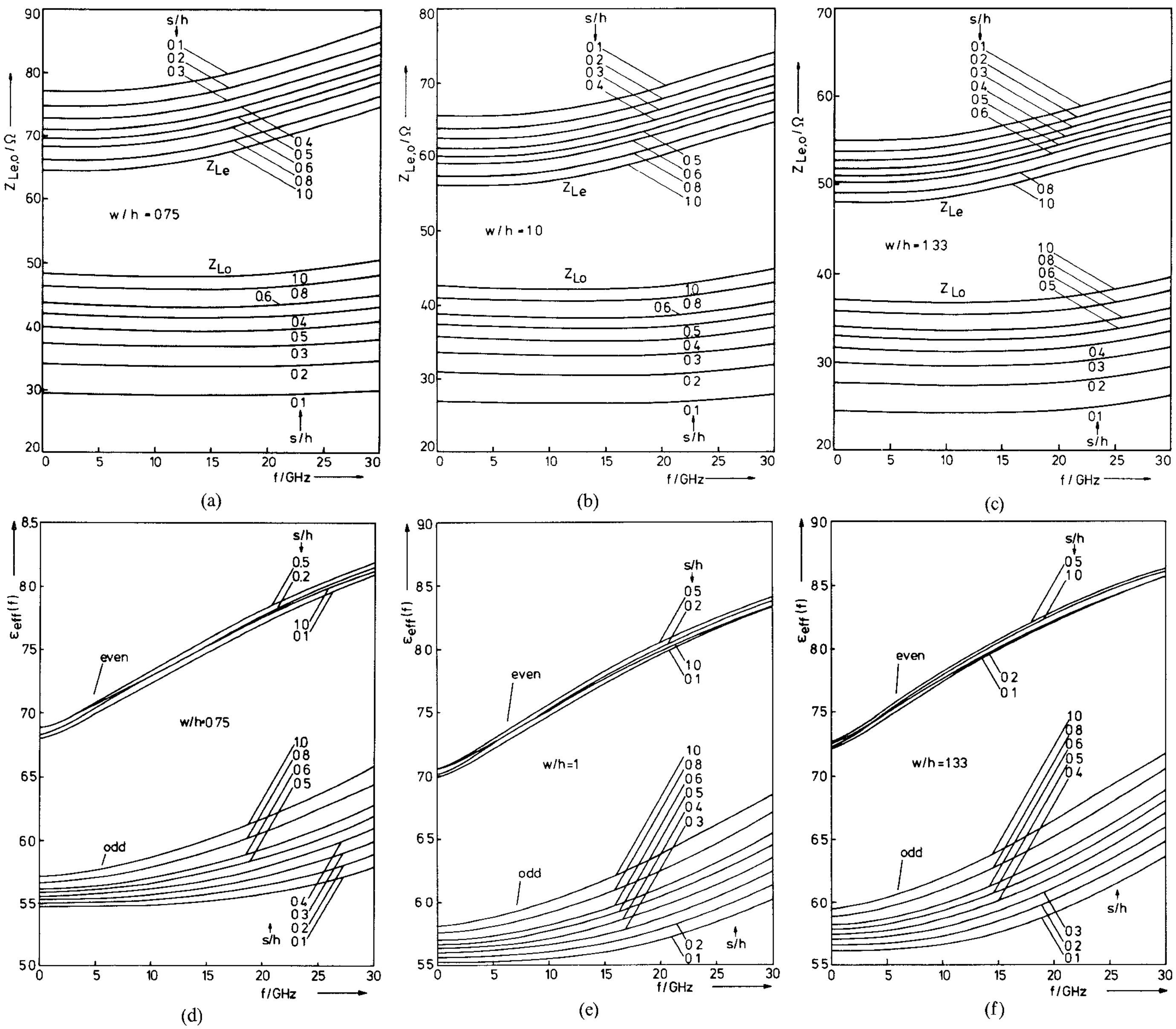


Fig. 3. The frequency dependent even- and odd-mode effective dielectric constants and characteristic impedances of coupled microstrip lines on a ceramic substrate (Alumina, $\epsilon_r = 9.70$, $h = 0.64$ mm). (a), (d) $w/h = 0.75$. (b), (e) $w/h = 1.0$. (c), (f) $w/h = 1.33$.

normalized line spacing $g = s/h$ as a parameter, these curves allow interpolation to obtain frequency-dependent even- and odd-mode impedance values for design.

Analytical expressions for the even- and odd-mode open-end equivalent lengths of the coupled microstrip section [37] are also reproduced here for completeness. The meaning of these quantities is illustrated in Fig. 1 and is analogous to the single microstrip open-end effect [32]. Their application for improved filter and coupler design has been outlined in a separate paper [37]. As for the single microstrip end effect, the inclusion of frequency dependence is not necessary for most design applications up to about 18 GHz [32], [34], and [35], and often even beyond that. In detail, the modal coupled microstrip equivalent end-effect lengths are described by

$$\Delta l_e = \left(\Delta l(2u, \epsilon_r) - \Delta l(u, \epsilon_r) + 0.0198 \cdot h \cdot g^{R_1}\right) \cdot \exp\left(-0.328 g^{2.244}\right) + \Delta l(u, \epsilon_r) \quad (12)$$

with

$$R_1 = 1.187 \cdot \left(1 - \exp\left(-0.069 u^{2.1}\right)\right)$$

and

$$\Delta l_o = \left(\Delta l(u, \epsilon_r) - h \cdot R_3\right) \cdot \left(1 - \exp\left(-R_4\right)\right) + h \cdot R_3 \quad (13)$$

with

$$R_2 = 0.343 \cdot u^{0.6187} + \left(0.45\epsilon_r/(1+\epsilon_r)\right) \cdot u^{(1.357 + 1.65/(1+0.7\epsilon_r))}$$

$$R_3 = 0.2974 \cdot (1 - \exp(-R_2))$$
$$R_4 = (0.271 + 0.0281\epsilon_r) \cdot g^{(1.167\epsilon_r/(0.66+\epsilon_r))} + (1.025\epsilon_r/(0.687+\epsilon_r)) \cdot g^{(0.958\epsilon_r/(0.706+\epsilon_r))}.$$

The end-effect quantities $\Delta l(u, \epsilon_r)$ and $\Delta l(2u, \epsilon_r)$, without subscript, represent single-line values for widths w and $2w$, respectively. The range of applicability is again defined by (1). The associated accuracy is 5 percent compared to the numerical hybrid-mode data basis [34], [35] employed for a fixed frequency of 4 GHz (actually, there is a slight increase with frequency). This should be accurate enough for most microstrip design purposes, since the length corrections (12), (13) themselves seldom contribute to the electrical length of a coupled microstrip section by more than 10–15 percent. It is observed that the even-mode equivalent end-effect length Δl_e decreases asymptotically to that of the corresponding single line Δl, if the spacing s/h is increased. Oppositely, the odd-mode length correction Δl_o approaches the single-line value Δl from below.

Coupled microstrip loss has not been considered here. The physical parameters required for such a computation, like surface roughness, dielectric loss tangent, and sheet resistivity, are not always known with good accuracy in practice. Therefore, approximate calculations, as found in several publications on the topic [10], [22], [24]–[28], seem adequate for many microstrip design cases. Also, sensitivity data for coupled microstrip lines has not been presented here, but is available in the technical literature [26], [38]–[40], or can be computed by the designer from the closed-form expressions provided here.

III. Conclusion

Novel frequency-dependent analytical expressions have been reported for the even- and odd-mode characteristics of parallel coupled microstrip lines. They are given in a form such that they can be implemented easily on a desktop computer or a programmable pocket calculator. In terms of their accuracy, which is specified for a very wide range of validity up to millimeter-wave frequencies, the described closed-form equations represent a considerable improvement to the foundations of coupled microstrip filter and coupler design. The equations are primarily set up for use in computer-aided microstrip circuit optimization routines, and are fully compatible with the needs and trends of modern computer-aided microwave integrated circuit design.

References

[1] T. G. Bryant, and J. A. Weiss, "Parameters of microstrip transmission lines and of coupled pairs of microstrip lines," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-16, pp. 1021–1027, 1968.

[2] ____, "MSTRIP (parameters of microstrip). Comp. Prog. Des.," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-19, pp. 418–419, 1971.

[3] G. Kowalski and R. Pregla, "Dispersion characteristics of single and coupled microstrips," *Arch. Elek. Übertragung (AEÜ)*, vol. 26, pp. 276–280, 1972.

[4] M. K. Krage and G. I. Haddad, "Frequency-dependent characteristics of microstrip transmission lines," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-20, pp. 678–688, 1972.

[5] R. Pregla and G. Kowalski, "Simple formulas for the determination of the characteristic constants of microstrips," *Arch. Elek. Übertragung (AEÜ)*, vol. 28, pp. 339–340, 1974.

[6] R. H. Jansen, "Computer analysis of edge-coupled planar structures," *Electron. Lett.*, vol. 10, pp. 520–522, 1974.

[7] J. B. Knorr and K. D. Kuchler, "Analysis of coupled slots and coplanar strips on dielectric substrate," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-23, pp. 541–548, 1975.

[8] J. B. Davies and D. Mirshekar-Syahkal, "Spectral domain solution of arbitrary coplanar transmission line with multilayer substrate," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-25, pp. 143–146, 1977.

[9] R. H. Jansen, "Fast accurate hybrid mode computation of nonsymmetrical coupled microstrip characteristics," in *Proc. 7th European Microwave Conf.*, (Copenhagen, Denmark), 1977, pp. 135–139.

[10] ____, "High-speed computation of single and coupled microstrip parameters including dispersion, higher-order modes, loss and finite strip thickness," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-26, pp. 75–82, 1978.

[11] ____, "Unified user-oriented computation of shielded, covered and open planar microwave and millimetre-wave transmission line characteristics," *Proc. Inst. Elec. Eng.*, vol. MOA-3, pp. 14–22, 1979.

[12] D. Mirshekar-Syahkal and J. B. Davies, "Accurate solution of microstrip and coplanar structures for dispersion and for dielectric and conductor losses," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-27, pp. 694–699, 1979.

[13] R. H. Jansen, *Microstrip Computer Programs — Short Descriptions.* Aachen, W. Germany: Verlag H. Wolff, 1981.

[14] A. Schwarzmann, "Microstrip plus equations adds up to fast designs," *Electronics*, pp. 109–112, Oct. 2, 1967.

[15] W. J. Getsinger, "Dispersion of parallel-coupled microstrip," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-21, pp. 144–145, 1973.

[16] B. Easter and K. C. Gupta, "More accurate model of the coupled microstrip line section," *Proc. Inst. Elec. Eng.*, vol. MOA-3, pp. 99–103, 1973.

[17] H. J. Carlin and P. P. Civalleri, "A coupled-line model for dispersion in parallel-coupled microstrips," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-23, pp. 444–446, 1975.

[18] S. Akhtarzad *et al.*, "The design of coupled microstrip lines," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-23, pp. 486–492, 1975.

[19] S. D. Shamasundara and N. Singh, "Design of coupled microstrip lines," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-25, pp. 232–233, 1977.

[20] J. Zehntner, "Analysis of coupled microstrip lines," *Microwave J.*, pp. 82–83, May 1978.

[21] A. E. Ros, "Design charts for inhomogeneous coupled microstrip lines," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-26, pp. 394–400, 1978.

[22] R. Garg and I. J. Bahl, "Characteristics of coupled microstrip lines," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-27, pp. 700–705, 1979.

[23] J. H. Hinton, "On design of coupled microstrip lines," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-28, p. 272, 1980.

[24] E. Hammerstad and O. Jensen, "Accurate models for microstrip computer-aided design," in *IEEE MTT-S Int. Microwave Symp. Dig.* (Washington, DC), 1980, pp. 407–409.

[25] S. March, "Microstrip packaging: Watch the last step," *Microwaves*, pp. 83–84, 87–88, 90, and 92–93, Dec. 1981.

[26] K. C. Gupta *et al.*, *Microstrip Lines and Slotlines.* Dedham, MA: Artech House, 1979, pp. 303–358.

[27] T. C. Edwards, *Foundations for Microstrip Circuit Design.* Chichester: Wiley, 1981, pp. 129–158, 221–230.

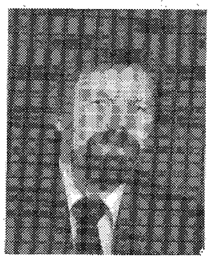
[28] K. C. Gupta *et al.*, *Computer-Aided Design of Microwave Circuits.* Dedham MA: Artech House, 1981, pp. 76–85, 120–126.

[29] R. H. Jansen and N. H. L. Koster, "New aspects concerning the definition of microstrip characteristic impedance as a function of frequency," in *IEEE MTT-S Int. Microwave Symp. Dig.* (Dallas, TX), 1982, pp. 305–307.

[30] W. J. Getsinger, "Measurement of the characteristic impedance of microstrip over a wide frequency range," in *IEEE MTT-S Int. Microwave Symp. Dig.* (Dallas, TX), 1982, pp. 342–344 and assoc. manuscript.

[31] R. H. Jansen and M. Kirschning, "Arguments and an accurate model for the power-current formulation of microstrip characteris-

tic impedance," *Arch. Elek. Übertragung (AEÜ)*, vol. 37, pp. 108–112, 1983.
[32] M. Kirschning *et al.*, "Accurate model for open end effect of microstrip lines," *Electron. Lett.*, vol. 17, pp. 123–125, 1981.
[33] M. Kirschning and R. H. Jansen, "Accurate model for effective dielectric constant of microstrip with validity up to millimetre-wave frequencies," *Electron. Lett.*, vol. 18, pp. 272–273, 1982.
[34] R. H. Jansen, "Hybrid mode analysis of end effects of planar microwave and millimeterwave transmission lines," *Proc. Inst. Elec. Eng.*, Pt. H, vol. 128, pp. 77–86, 1981.
[35] R. H. Jansen and N. H. L. Koster, "Accurate results on the end effect of single and coupled microstrip lines for use in microwave circuit design," *Arch. Elek. Übertragung (AEÜ)*, vol. 34, pp. 453–459, 1980.
[36] J. S. Hornsby, "Full-wave analysis of microstrip resonator and open-circuit end effect," *Proc. Inst. Elec. Eng.*, Pt. H, vol. 129, pp. 338–341, 1982.
[37] M. Kirschning *et al.*, "Coupled microstrip parallel gap model for improved filter and coupler design," *Electron. Lett.*, vol. 19, pp. 377–379, 1983.
[38] R. J. Roberts, "Effect of tolerances on the performance of microstrip parallel-coupled bandpass filters," *Electron. Lett.*, vol. 7, pp. 255–257, 1971.
[39] B. Ramo Rao, "Effect of loss and frequency dispersion on the performance of microstrip directional couplers and coupled filters," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-22, pp. 747–750, 1974.
[40] C. Gupta, "Design of parallel coupled line filter with discontinuity compensation in microstrip," *Microwave J.*, vol. 22, pp. 39–40, 42–43, 57, 1979.



Manfred Kirschning was born in Mölln, West Germany, on May 27, 1951. He received the Dipl.-Ing. degree in electrical engineering in 1977 from the Technical University, Hannover, West Germany. From 1977 through 1981, he was enrolled as a student of industrial management and economics at the Technical University, Aachen, West Germany, where he received the Dipl.-Wirtschaftsing. degree in 1981. He will receive the Dr.-Ing. degree in electrical engineering from Duisburg University, West Germany, in February, 1984.

From 1977 to 1978, he was a Research Engineer in the Institute of Radio Frequency Techniques, RWTH Aachen, where he was engaged in research in the field of coupled microstrip filters and coupler analysis and synthesis. From 1978 to 1983, he was employed as a Research Scientist in the department of electrical engineering at Duisburg University, West Germany, where he worked in the areas of microwave theory, CAD techniques, modeling techniques of microstrip discontinuities, permittivity measurements, and filter and coupler synthesis on desktop computers. Since April of 1983, he has been a Marketing Manager, Honeywell GmbH, Maintal, West Germany.

Rolf H. Jansen (M'75) was born in Cologne, West Germany, in 1946. He received the M.S. and Ph.D. degrees, both in electrical engineering, from RWTH Aachen in Aachen, West Germany, in 1972 and 1975, respectively. His Ph.D. thesis was on the numerical analysis of arbitrarily shaped microstrip structures.

From 1972–1976, he was Research Assistant at the RWTH Aachen microwave laboratory. From 1976 to 1979, he was a Senior Research Engineer in the same laboratory, working on the frequency-dependent characterization of planar structures and the CAD of microwave circuits, and also had responsibility for the thin-film technology of the laboratory. Since 1977, he has been a scientific consultant and avocational staff member of Standard Elektrik Lorenz AG (SEL), Pforzheim, W. Germany in the Radio Communication Division. Since 1979, he has been an Associate Professor at Duisburg University, engaged in teaching and research in the microwave field and the computer-aided design of microwave broad-band components, like FET-amplifiers and mixers. For the year 1981/82, he stayed with SEL, Pforzheim, as a full-time staff member. He is the author and coauthor of about 40 technical papers.

Dr. Jansen is a member of VDE, the German association of electrical engineers, and the recipient of the outstanding publications award in 1979 of the German Society of Radio Engineers.